MEASUREMENT OF THERMOPHYSICAL PARAMETERS OF HUMAN HAIR AND OTHER FIBERS. MATHEMATICAL MODELING

[1]N.G. Kokodii, [2]S.V. Pogorelov, [2]V.A. Timaniuk, [2]I.V. Krasovskyy, [1]D.V. Gurina, [1]I.V. Garyachevskaya, [1]D.O. Protector
[1]Karazin Kharkiv National University, Kharkiv, Ukraine
[2]National University of Pharmacy, Kharkiv, Ukraine
Email:kokodiyng@gmail.com

Abstract: The thermal parameters of a thin fiber (rod) can be found by examining the process of its heating and solving the inverse problem of heat conductivity. With the help of mathematical modeling, algorithms for determining thermal conductivity, heat capacity, and thermal diffusivity can be developed.
Keywords: fiber, thin rod, thermal parameters, measurement, mathematical modeling.

INTRODUCTION

By measuring the process of heating a human hair or other similar fiber and solving the inverse problem of thermal conductivity, you can find the values of its thermal parameters - thermal conductivity, heat capacity, heat diffusivity.

We investigate this problem by applying mathematical modeling. Let us first solve the direct problem of heating a thin rod. Then, assuming that these data were obtained in the experiment, we will use them to find the thermal parameters of this rod.

Let's solve the heat conduction equation for a thin rod [1]:

$$\frac{\partial^2 T(z,t)}{\partial z^2} - \gamma^2 T(z,t) - \frac{1}{a}\frac{\partial T(z,t)}{\partial t} = 0. \qquad (1)$$

Here $\gamma^2 = \frac{4\alpha_p}{\pi k D^2}$, T – temperature, D – rod diameter, a = k/(cρ) – heat diffusivity, k – thermal conductivity, c – specific heat capacity, ρ – density, α_p – heat transfer coefficient per unit length of the rod. For thin rods, it does not depend on the diameter [2].

The second term describes the heat transfer from the side surface.

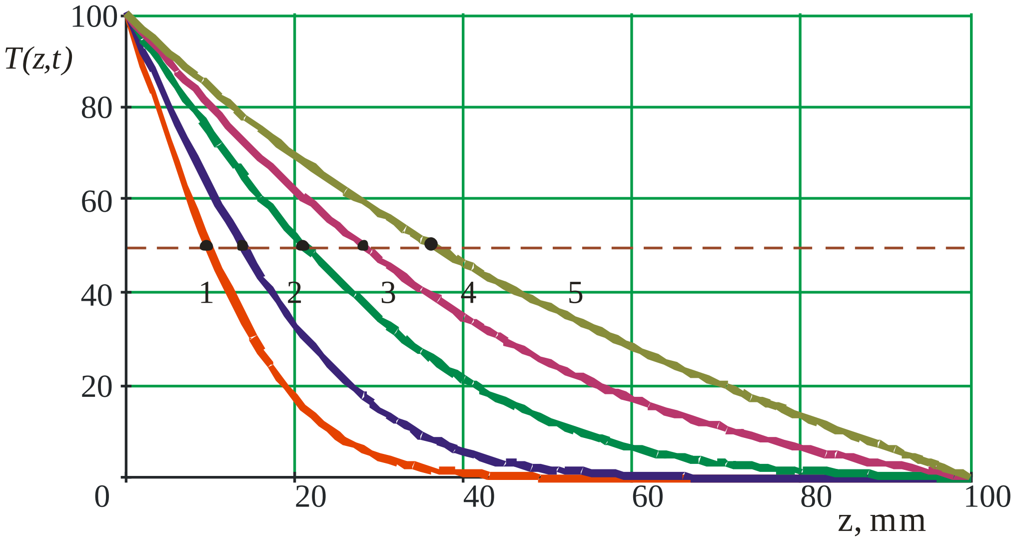
Initial condition is *T*(0,t) = 0.

One end of the rod is heated - T(0,t) = T_0, the other end has an ambient temperature - T(L,t) = 0. Here L is the length of the rod.

Equation (1) was solved for the following data: rod length L = 100 mm, diameter D = 0.5 mm, density ρ= 8960 kg/m^3, thermal conductivity k = 398

W/(m*K), heat capacity c = 400 J/(kg*K), heat transfer coefficient a_p= 0.02 W/(m*K).

On fig. 1 shows the temperature distribution along the rod at different times. It can be seen that the rod gradually warms up along its length.

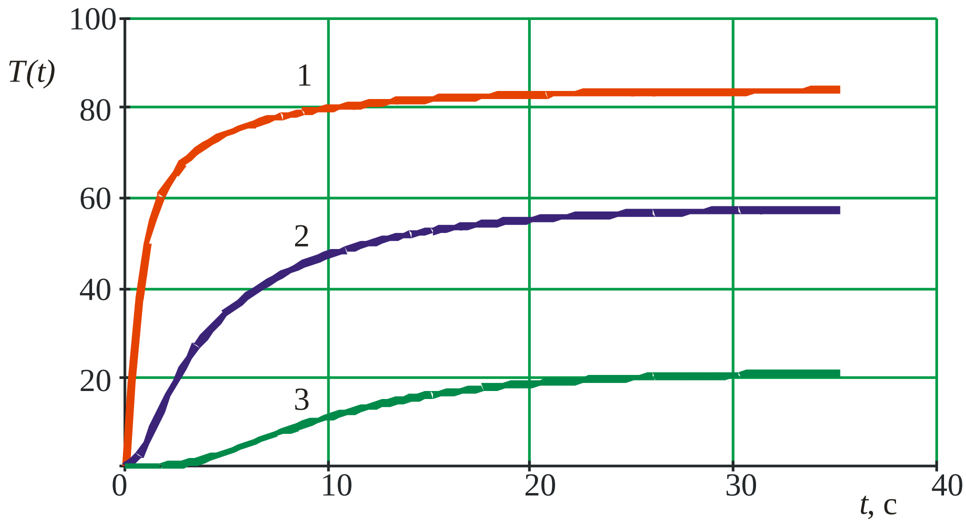


Rice. 1. Temperature distribution along the rod
1 - t = 1 s, 2 - t = 2 s, 3 - t = 5 s, 4 - t = 10 s, 5 - t = 30 s

An important characteristic is the heating time constant τ. It is known from the theory of thermal conductivity [1] that it is determined by the ratio between the heat capacity and heat exchange with the environment. For a thin rod uniformly heated along its length

$$\tau = \frac{\pi D^2 c \rho}{4\alpha_p} = 35.2 \text{ s}. \qquad (2)$$

It determines the temperature settling time in the rod. Usually it is considered equal to 4 τ.But in fig. 2 shows that it is much smaller, depends on the *z* coordinate, and is determined by the time of heat propagation from the heating source to a given point. This is due to uneven heating of the rod.



Rice. 2. Change in temperature at different points of the rod
1 - z = 10 mm,   2 - z = 30 mm,   3 - z = 70 mm.

1. MEASUREMENT OF HEAT DIFFUSIVITY

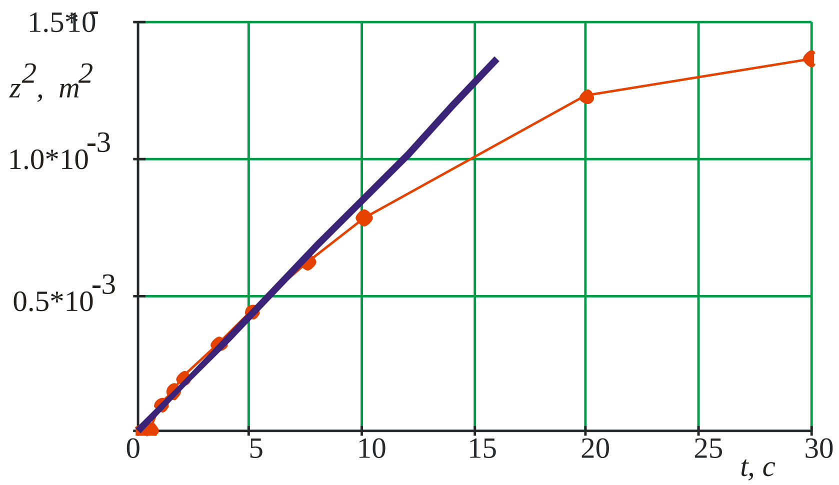
The thermal diffusivity characterizes the speed of propagation of the thermal front in the medium. On fig. 1 shows the temperature distribution in the rod at different times. The position of the thermal front is characterized by the coordinate where the temperature is 2 times less than the maximum. This is indicated by dots on the curves T(z,t). Over time, the heat front moves along the rod.

The position of the thermal front *z* at time t is described by the Einstein-Smoluchowsky formula [1]

$$z = \sqrt{at}\,, \qquad (3)$$

where *a* is the thermal diffusivity.
It can be found from the dependence z(t).

On fig. 3 shows a plot of $z^2 = f(t)$. It should be a straight line with a slope a.



Rice. 3. Movement of the thermal front

A linear dependence exists at the initial moments of time, when thermal processes are far from the stationary state. The straight line in fig. 3 is built using the least squares method. The value of the thermal diffusivity a = 8.5*10^{-5} m^2/s.

2. MEASUREMENT OF THERMAL CONDUCTIVITY AND HEAT CAPACITY

For stationary mode ($\frac{\partial}{\partial t} = 0$) equation (1) looks like this:

$$\frac{d^2 T(z,t)}{dz^2} - \gamma^2 T(z,t) = 0 .$$

His solution is:

$$T(z) = T_0 \frac{\mathrm{sh}\left(\gamma\left(L - z\right)\right)}{sh\left(\gamma L\right)} . \quad (4)$$

On fig. 1 is line 5 fort = 30 s.

Using the method least squares, it is found that γ= 14.9 1/m. Then from the formula for γ follows that

$$k = \frac{4\alpha_p}{\pi D^2 \gamma^2} = 459 \frac{W}{m \cdot K} .$$

Coefficient heat diffusivity was found earlier:

$$a = 8.5*10^{-5} \text{ m}^2/\text{s}.$$

Therefore, the volume heat capacity can be found:

$$c\rho = \frac{k}{a} = 5.4 \cdot 10^6 \frac{J}{m^3 \cdot K} .$$

If the density is known, the specific heat capacity can be found. For density ρ= 8960 kg/m^3

$$c = 603 \text{ J/(kg * K)}.$$

CONCLUSION

Mathematical modeling of the heating process of a thin rod is made. The inverse problem of heat conduction is solved. The thermal parameters of the rod are found - thermal conductivity, heat capacity, heat diffusivity. The proposed method can be used to solve problems in physics and biophysics.

REFERENCES

1. H.S. Carslaw, J.C. Jaeger. Conduction of heat in solids. Oxford Press, 1960, 488 p.
2. R.C.L. Bosworth. Heat transfer phenomena. The flow of heat in physical systems. NY, 1956, 276 p.